A METHOD OF CALIBRATING HELMHOLTZ COILS FOR THE MEASUREMENT OF PERMANENT MAGNETS

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ABSTRACT
The Helmholtz coil configuration is often used to generate a uniform magnetic field in space, and may also be used to measure the magnetic moment of bar and plate magnets. In the past, Helmholtz coils used for measurement have been calibrated by the use of standard magnets. A method is presented here for calibrating these coils using a known current source and gaussmeter, which should be easier and result in greater accuracy.

1. INTRODUCTION

Manufacturers and users of permanent magnets need to measure the strengths of large numbers of magnets by means which are quick, accurate, and require a minimum of calculation and labor. Many (but not all) magnets may be measured in this way by the use of a Helmholtz coil together with an integrating fluxmeter. Helmholtz coils used in this manner in the past have usually been calibrated by use of standard magnets. Since permanent magnet materials are not perfectly homogeneous in their magnetic properties (and may vary with temperature and age, as well), and since NIST (the former National Bureau of Standards) does not maintain magnetic standards, such magnets may be difficult to obtain or confirm to sufficient accuracy. The method suggested in this paper is an indirect one, but is easily accomplished and enables coil calibration by means which may be known to high accuracy.

A Helmholtz coil is actually a pair of similar coils with equal numbers of turns. The coils are short, thin-walled rings, and are mounted coaxially at a distance of one coil radius from each other. When the coils are connected (preferably in series) and current is passed through them, a highly uniform magnetic field is produced in a considerable volume of space between the two coils. At the center, the first three derivatives of field strength with position all go to zero in every direction (reference 2). It is this property, plus the fact that the region of uniform field is accessible from almost every direction, which gives the

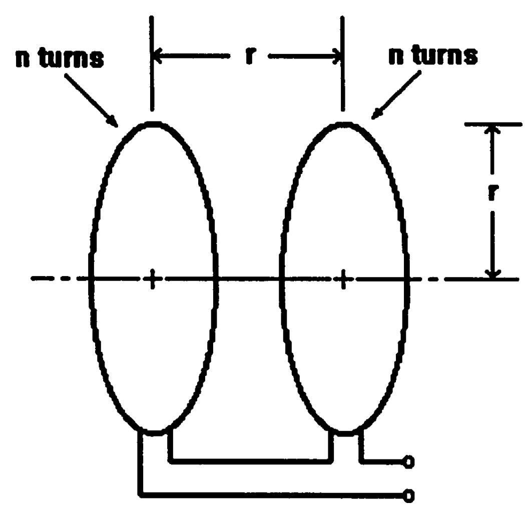


Fig.1: Helmholtz Coil

Helmholtz construction its special usefulness.

The type of magnet measurable by a Helmholtz coil is one which can be represented by a magnet dipole, or assembly of dipoles with a common axial direction. A magnetic dipole, as shown in figure 2, consists of two magnetic poles of equal and opposite strength, separated by a distance l_p. The North pole is a magnetic flux source, radiating straight lines of flux outward with equal distribution in all direction. The opposite South pole is a flux sink, into which flux disappears at the same density per solid angle, from all direction. Since the surface of a sphere is equal to $4\pi r^2$, a single magnetic pole of strength 4π produces a unit magnetic field at unit radius. The magnetic moment of the dipole is equal to the pole strength (in units of flux density times area, or flux) times the pole spacing l_p. For a real magnet, this is equal to the integral of flux density times a differential of volume, over the entire volume of the magnet. Long, thin magnets are reasonably well represented by a magnetic dipole, except close to the magnet, and plate magnets may be approximated by a series of dipoles with a common direction. With suitable corrections, arc segments may also be represented in this way (reference 1). Cylindrical magnets, such as those used in some brushless DC motors and rotary linear actuators, cannot be measured by this technique because the magnetic axes do not lie in the same direction, and cancel partially or completely.

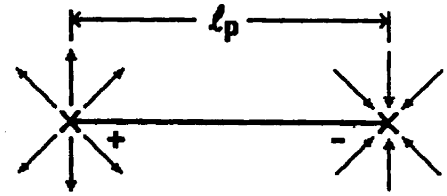
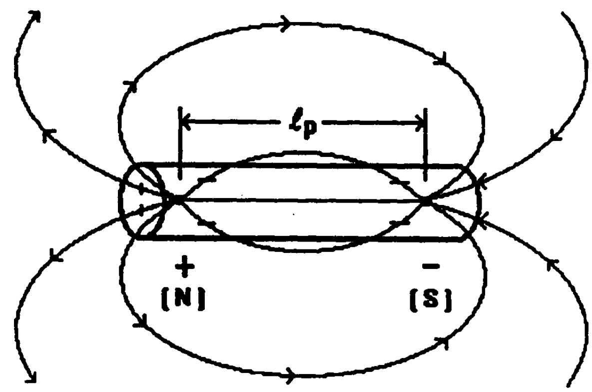
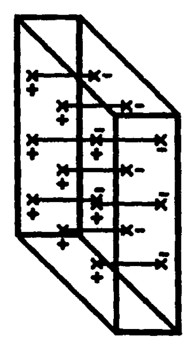


Figure 2: Magnetic Dipole



3a: Bar Magnet Represented as a Magnetic Dipole



3b: Representation of a Slab Magnet as a Set of Magnetic Dipoles

Figure 3

When a Helmholtz assembly is used to measure a magnet, the magnet is placed between the coils in the region of uniform field, with the magnetic axis parallel to the coil axis, and the fluxmeter attached to the coils is zeroed. The magnet is then withdrawn far enough from the coils that its effect on them is negligible. The reading on the fluxmeter times the coil constant is then equal to the magnetic moment of the tested part. Another method is to rotate the magnet

180°. The fluxmeter reading times the coil constant is then equal to twice the magnetic moment of the part.

If magnetic flux Φ linking an electrically conductive coil of n turns changes with time, a voltage is developed across the coil according to the Faraday induction law,

$$E = n\frac{d\Phi}{dt} \qquad (1)$$

The fluxmeter reading is proportional to the integral of voltage with time,

$$Q = k_m \int E\,dt = k_m \int n\left(\frac{d\Phi}{dt}\right)dt = k_m n\Phi + C \qquad (2)$$

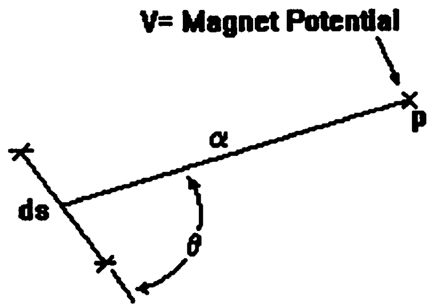
In the above, the constant k_m is a scale multiplier which may be set on the instrument, and the constant C is the offset, normally set to zero before the reading is made.

Fluxmeters are often used together with a search coil of known area and number of turns, to determine the average magnetic flux density component normal to the plane of the coil, as an averaging gaussmeter. They may also be used to determine the total magnetic flux crossing a section within a magnet or magnetically permeable pole structure, at locations which would not be accessible to a Hall-type gaussmeter. For this purpose a close-fitting coil is wrapped around the part and connected to the fluxmeter, and the instrument is then zeroed. The coil is then removed from the part, without disconnecting it from the fluxmeter, and taken far enough that the magnetic circuit has no noticeable effect on the meter. The fluxmeter reading is then proportional to the total flux crossing the section. It is important that the coil be as closely fitting as possible, to avoid errors due to flux leakage. A correction is sometimes applied for the sense wire thickness. When measurements are made with a Helmholtz coil, on the other hand, there is free space around the part, and handling is much easier.

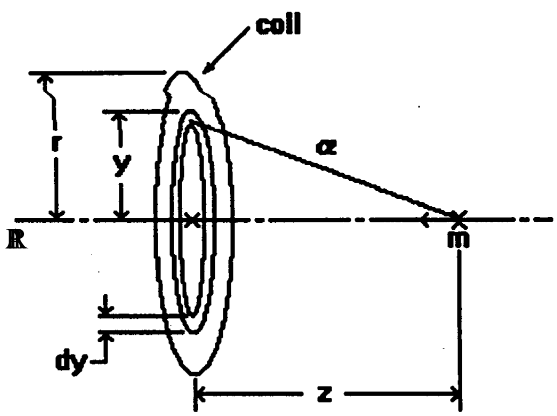
By comparing the units of magnetic moment (magnetic flux density times length cubed) to those of the fluxmeter output (magnetic flux density times length squared), it is apparent that the Helmholtz coil calibration constant will include a unit of length.

If the coil windings have negligibly small winding thickness and axial length and if the coils are perfectly constructed and aligned, the coil constant may be derived from its dimensions and number of turns. If the device is to have a coil constant large enough to be useful without excessive resistance, however, the winding dimensions will probably be large enough to affect the measurement accuracy. It is possible to correct the calculation to account for these dimensions (reference 2). Other errors, however, are more difficult to handle, such as misalignment, variations in winding, coil spacing errors, out-of–roundness of the coils, etc. Because of these, it is best to measure the coil constant by some means, such as the use of standard magnets or the method described here.

2. FLUX FROM A SMALL MAGNETIC DIPOLE LINKING A HELMHOLTZ COIL



4a: Magnetic Dipole at Distance α From Point p



4b: Magnetic Dipole Aligned with Coil Axis

Figure 4

The magnetic potential of a magnetic dipole distant from the point of measurement is:

$$V = \frac{m}{4\pi\alpha^2}\cos\theta \qquad (3)$$

with $\cos\theta = \frac{z}{\alpha}$

and

$$\alpha = \sqrt{y^2 + z^2}$$

$$V = \frac{m}{4\pi}\frac{z}{(y^2+z^2)^{3/2}} \qquad (4)$$

$$B_z = -\frac{\partial V}{\partial z} = \frac{m}{4\pi}\left(\frac{1}{(y^2+z^2)^{3/2}} - \frac{3z^2}{(y^2+z^2)^{5/2}}\right) \qquad (5)$$

The flux linking each coil is

$$\phi_c = n\int B_z 2\pi\, y dy \qquad (6)$$

Substituting for B_x and integrating,

$$\Phi = mn\left(\frac{1}{(y^2+z^2)^{1/2}}\right) - \frac{z^2}{(y^2+z^2)^{3/2}}\Bigg|_0^{y=r} \qquad (7)$$

for $y = r$, $z = \frac{r}{2}$,

$$\Phi = \frac{mn}{r}(.71554175...) \qquad (8)$$

3. MAGNETIC FIELD AT A POINT ON THE AXIS CAUSED BY FIELD CURRENT

On the other hand, for the condition of electric current passing through a coil of the same construction, in reference to a point p on the coil axis at a distance from the coil plane (figure 5), Amperes law states that for a differential of coil length ds carrying current i at a distance α

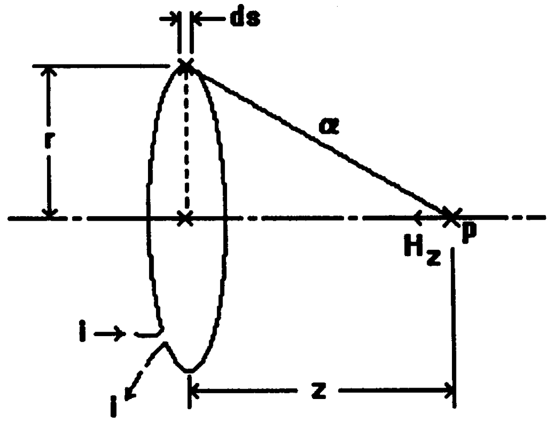


Figure 5: Field H Caused at Point p by Current in Coil Length Element ds

from point p, an increment of magnetic coercive force dH is caused at p according to:

$$dH = \frac{i}{4\pi} \frac{\overline{ds} \times \overline{\alpha}}{|\alpha|^3} \qquad (9)$$

since:

$$\alpha = \left(z^2 + r^2\right)^{1/2}$$

The component of dH in the axial direction is:

$$dH_z = \frac{i}{4\pi} \frac{r\,ds}{\left(z^2 + r^2\right)^{3/2}} \qquad (10)$$

which may be integrated to:

$$H_z = \frac{n i r^2}{2\left(z^2 + r^2\right)^{3/2}} \qquad (11)$$

(per coil)

at z = r / 2, for two coils aiding, each of n turns,

$$H_c = ni \frac{r^2}{\left[r^2 + \left(\frac{r}{2}\right)^2\right]^{3/2}} \qquad (12)$$

$$\frac{H_c}{i} = \frac{n}{r}(.71554175) \qquad (13)$$

4. COIL CONSTANT

Dividing equation (8) by equation (13) and arranging,

$$m = \Phi\left(\frac{i}{H_c}\right) \qquad (14)$$

The coil calibration constant, then, is:

$$k_c = \left(\frac{i}{H_c}\right) \qquad (15)$$

The coil current i may be determined to high accuracy by means traceable to NIST. Magnetic flux density may be measured by use of a gaussmeter. Some gaussmeters (such as those based on nuclear magnetic resonance) are capable of excellent accuracy. Another possibility would be to measure the flux crossing a coil of known area and turns by use of the same fluxmeter used for the moment measurements, thus canceling any scale error which might be present.

Measurements at our lab, made with instruments of somewhat limited accuracy, confirm equations to an accuracy of about .5% with a total uncertainty of measurement from all sources of about .7% maximum. Reference 1 quotes data confirming equation (8) to about

.35%.

5. DIRECT METHOD

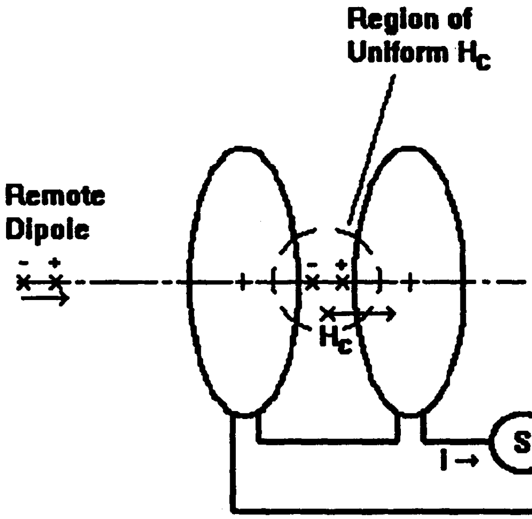


Figure 6: Movement of Dipole from ∞ into Uniform Magnetic Region along the Coil Axis

The coincidence of these two integrals, equation (8) and (13), suggests that a more direct method of calculation might be possible. Such is indeed the case.

A unit pole experiences unit mechanical force when acted upon by a magnetic field of unit intensity H. An electric current passing through the two coils in series would produce some distribution of magnetic field in space around the coil assembly, and a uniform field of intensity H_c in a volume of space between the coils. If a magnetic dipole, consisting of two equal and opposite poles of strengths $\pm \Phi$, separated by distance l_p, is brought from infinity into the region of uniform field along the coil axis, mechanical work will be done on each pole by the magnetic field. Since each pole traversed the same path, the work done on one pole exactly cancels the work done on the other pole, of opposite sign, except for the path difference l_p, within the region of uniform field of strength H_c. The net mechanical work, therefore, must be:

$$W_{mechanical} = H_c \, \Phi \, l_p \qquad (16)$$

To simplify the argument, let it be imagined that the coils are made of superconducting material, with no electrical resistance (we could just as well add in a constant resistance term, and then subtract it out later). Let it also be assumed that the power source emits constant current i. With the pole pair at infinity, the voltage E in the circuit is zero, because of the superconductivity of the coil. As the dipole is now brought towards the coil pair, a voltage will be induced in the winding due to lines of magnetic flux cutting the coils, and work will be done by the source:

$$W_{source} = \int E i \, dt \qquad (17)$$

With no losses, due to the superconductivity of the coil, the only possible source of the work done must be the source:

$$W_{mechanical} = W_{source} \qquad (18)$$

or:

$$H_c \Phi l_p = i \int E dt \qquad (19)$$

However, $\Phi \, l_p = m$, the magnetic moment of the dipole, and $\int E \, dt$ is recognizable as the fluxmeter reading (divided by the number of turns n and the meter constant k_m), and so:

$$m = \int E\,dt \left(\frac{i}{H_c} \right) \quad (20)$$

To obtain the meter constant, then,

A. Connect the coil pair to a current source, and apply an amount of current to the coils which is high enough to give a fluxmeter reading which is large enough for good accuracy, but which is not so high as too cause overheating of the coils. Measure the current by accurate means.

B. Measure the flux density B_x in the direction of the coil axis.

C. Divide the current i by the magnetic flux density B_x and correct units if required. If the fluxmeter reading is in maxwells, as is usual, and if the units of magnetic moment desired are to be in oersted-cm, the result should be multiplied by $4\pi / 10$ oe-cm to obtain the coil constant.

6. SOME PRACTICAL CONSIDERATIONS

When an integrating-type fluxmeter is used with a small, low electrical resistance coil, the coil resistance is small enough to have negligibly small effect on the fluxmeter reading, and is ignored. The winding resistance of the Helmholtz coil, on the other hand, may be large enough to influence the fluxmeter reading, especially on high-sensitivity settings. A correction formula should be available in the instrument manual or from the manufacturer. Other shapes of coils than round are sometimes used in the Helmholtz configuration, and other numbers of coils than two, but except for a few special applications their advantages seem to be more theoretical than practical. Square or rectangular coils may be used, but such coils are much more difficult to wind uniformly, and the coil support frames are more difficult to construct to the same accuracy as round shapes. Coil patterns requiring more than two coils are known (references 2 and 3) which result in a more uniform field or large useful volume, but access to the workspace becomes more restricted.

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